Long and Synthetic Division of Polynomials

Long and synthetic division are two ways to divide one polynomial (the dividend) by another polynomial (the divisor). These methods are useful when both polynomials contain more than one term, such as the following two-term polynomial: $x^2 + 3$. This handout will discuss the rules and processes for dividing polynomials using these methods.

Three Rules before Dividing Polynomials

There are a few rules to consider when dividing polynomials. First, the divisor must have a smaller power of x than the dividend. Just as five cannot be evenly divided by eight, $(x - 1)$ cannot be divided by $(x^3 - 1)$. The second rule specifies that both the divisor and dividend must be ordered by descending powers of x. Once the powers of x are properly ordered, the third and final rule states that any missing powers of x between the highest power and the constant number must be filled in with a 0. For example, a polynomial such as $(7x + 5x^3 + 8)$ must be rewritten as $(5x^3 + 0x^2 + 7x + 8)$.

Long Division

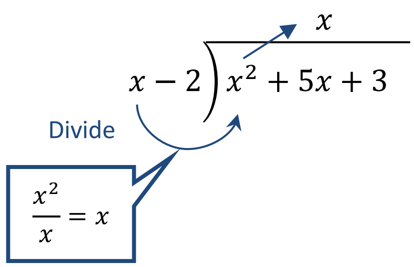
Long division is a reliable tool to divide any two given polynomials. The following example problem will explain the steps needed when using this method.

Example A: $(x^2 + 5x + 3) \div (x - 2)$

$$x - 2 \overline{\smash{\big)}\, x^2 + 5x + 3}$$

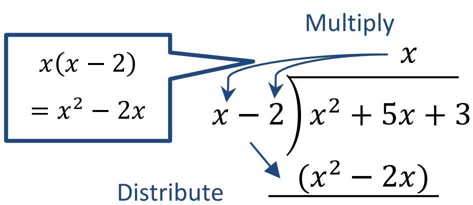
Step One

Write the problem in long division form. The dividend is placed inside of the long division symbol, and the divisor is placed to the left.



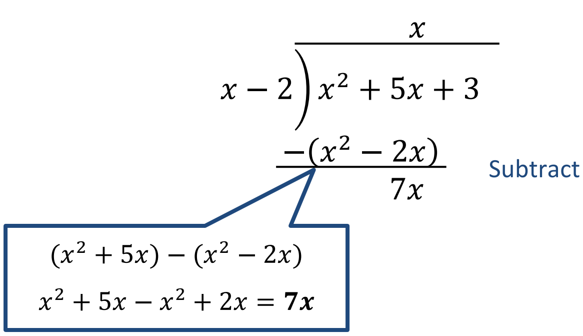
Step Two

Divide the first term in the dividend by the first term in the divisor. That is, $\frac{x^2}{x}=x$. Write the result, also known as the quotient, above the dividend, aligning like terms.



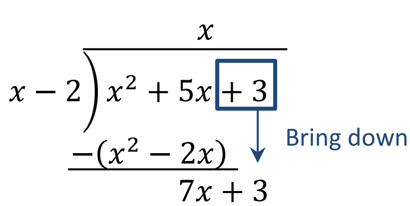
Step Three

Multiply the x from the previous step by each term in the divisor. Then, write the product under the dividend and align the terms of the product with the like terms of the dividend as shown.



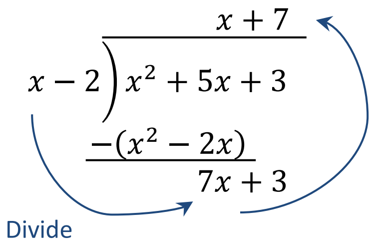
Step Four

Subtract x^2-2x from x^2+5x. To do this, distribute the negative sign, which will change $-(x^2-2x)$ to $-x^2+2x$. Write the result, $7x$, below the $2x$.



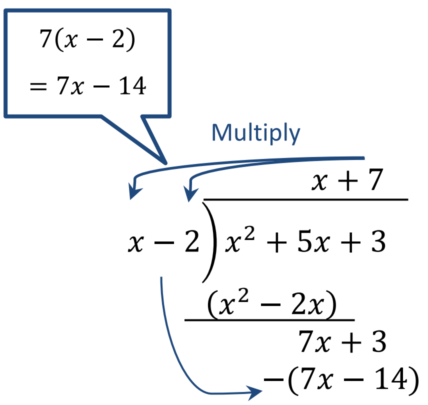
Step Five

Bring the $+\,3$ down from the dividend so that it is next to the $7x$. This forms a new dividend: $7x+3$.



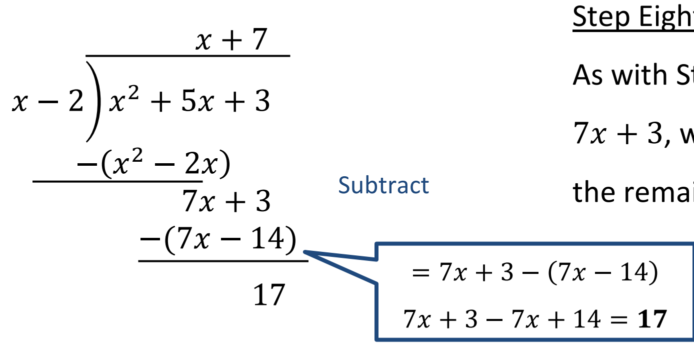
Step Six

Similar to Step Two, divide the first term in the new dividend by the first term in the divisor. That is $\frac{7x}{x}$, which equals 7. Then, write the quotient on top of the dividend, with like terms aligned.



Step Seven

Similar to Step Three, multiply the 7 from the previous step by each term in the divisor, and write the product under the new dividend. Align the terms of the product with the matching terms of the dividend.



Step Eight

As with Step Four, subtract $7x - 14$ from $7x + 3$, which results in 17. This result will be the remainder in the final solution.

$$x+7+\frac{17}{x-2}$$

Solution

The solution is the quotient $x + 7$, plus the remainder, 17, divided by $x - 2$.

Synthetic Division

Synthetic division is a shorthand method to divide polynomials. It is designed to make the division process faster once a person feels confident with long division. A problem that is written in general terms, such as $(Ax^5 + Bx^4 + Cx^3 + Dx^2 + \cdots) \div (x - a)$, is set up in the form seen below this paragraph. Each letter stands for a number; for example, the a comes from the divisor and the A, B, C, and D are the coefficients of each term from the dividend.

$$\begin{array}{c|ccccc} a & A & B & C & D & \ldots \\ & & & & & \\ \hline \end{array}$$

Unfortunately, there is one limitation that keeps this method from being a complete replacement for long division: the divisor must be in the form $(x - a)$. If the divisor has a higher degree of x than the dividend, such as $(x^2 + 5)$, or has more than two terms, then synthetic division would be more difficult to perform than long division.

Example B: $(x^2 + 5x - 3) \div (x + 2)$

Step One

Ensure that the divisor is in $(x - a)$ form, and the dividend is ordered by descending degrees of x.

$$(x^2 + 5x - 3) \div (x - (-2))$$

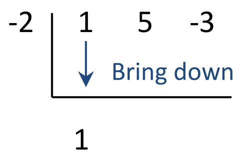
Step Two

Write the problem in synthetic division form.

$$\begin{array}{c|ccc} -2 & 1 & 5 & -3 \\ & & & \\ \hline \end{array}$$

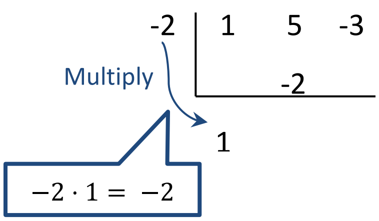
Step Three

Bring the first coefficient to the bottom row. This handout will refer to this row as the solution row.



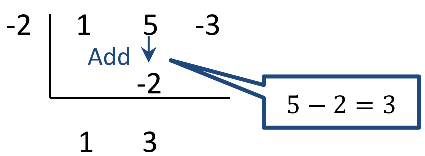
Step Four

Multiply the 1 in the solution row by the divisor. Then, write the product in the middle row of the second column below the 5.



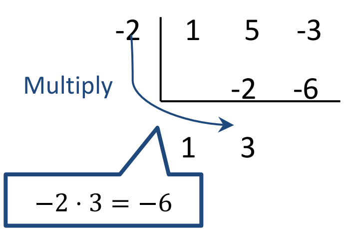
Step Five

Add the two numbers in the second column. Their sum will be 3. Write this result in the solution row of the same column.



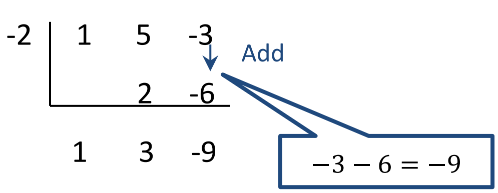
Step Six

Similar to Step Four, multiply the 3 in the solution row by the divisor, resulting in -6. Then write this product in the middle row of the third column below -3.



Step Seven

Similar to Step Five, add -3 and -6, which equals -9. Write the sum in the solution row below -6. This number is the answer's remainder because it is the last number in the solution row.



Step Eight

2	1	5	-3
		-2	-6
	1	3	-9

Rewrite

$1x + 3 + \frac{-9}{x+2}$

Since this example's remainder is not 0, it needs to be written over the original divisor $x + 2$ in fractional form. The final result for this example's remainder is $\frac{-9}{x+2}$.

Original dividend:

$1x^2 + 3x - 3$

After dividing:

$$1x + 3 + \frac{-9}{x+2}$$

One degree less than the original

The remaining numbers in the bottom row will form the quotient. The quotient is written starting with the first number on the left. This number will be written with an x that is one degree lower than the highest degree in the original dividend. In this case, the term with the highest degree in the dividend is $1x^2$, so the first term of the quotient should be $1x$. Each term following the first term will be written with an x that is one degree less than the previous term. In this case, the 3 will be written as $3x^0$, which can be simplified to just 3.

In summary, the first x will have a degree one less than the original dividend. Each term that follows will have an x that is one degree lower than the previous.

Solution

The solution is the combination of the quotient, $x + 3$, and the remainder, -9 over $x + 2$.

$$x + 3 - \frac{9}{x+2}$$

Practice Problems

Solve the following problems using both long division and synthetic division when possible. Make sure the answers match when both methods are used. Correct answers are on the next page.

1. $(2x^3 + 3x^2 + 5x + 9) \div (x - 20)$
2. $(2x^3 + 3x^2 + 4x + 5) \div (2x^2 + 3x + 4)$
3. $(2x^3 + 2) \div (x + 3)$
4. $(5x^3 - 25x^2 + 6x^4 - 30x^3) \div (x + 5)$
5. $(3x^2 + 5x + 10) \div (x - 5)$
6. $(2x^3 + 18x + 5x^2 + 45) \div (2x + 5)$
7. $(2x^2 + 5) \div (x - 29)$
8. $(x^4 + 7x^2 + 10) \div (x^2 + 2)$
9. $(x + 5) \div (x - 6)$
10. $(x^5 + 1) \div (x - 2)$

Answers

1: $2x^2 + 43x + 865 + \frac{17309}{x-20}$ (long and synthetic division)

2: $x + \frac{5}{2x^2+3x+4}$ (only long division)

3: $2x^2 - 6x + 18 - \frac{52}{x+3}$ (long and synthetic division)

4: $6x^3 - 55x^2 + 250x - 1250 + \frac{6250}{x+5}$ (long and synthetic division)

5: $3x + 20 + \frac{110}{x-5}$ (long and synthetic division)

6: $x^2 + 9$ (only long division)

7: $2x + 58 + \frac{1687}{x-29}$ (long and synthetic division)

8: $x^2 + 5$ (only long division)

9: $1 + \frac{11}{x-6}$ (long and synthetic division)

10: $x^4 + 2x^3 + 4x^2 + 8x + 16 + \frac{33}{x-2}$ (long and synthetic division)